Trigonometry and 3-Space

Sample Final Examination: Mathematics 122 version 2009-A

This examination paper was prepared by a committee of NB high school teachers,
as part of a workshop held at the University of New Brunswick, March 12 – 13, 2009.

Time: 2 hours.
Calculators: Scientific calculator required.
Graphing calculator permitted, at the teacher's discretion.
Formulae: see the last page.

- Please check that there are 13 pages, including this page.
- There are three parts to the test, Parts A (17 points), B (18 points) and C (25 points).
  Total points: 60.
- **Part A** appears on pages 2 to 4 and consists of 17 short problems.
  Write your answers clearly in the boxes or on the graphs provided.
  Answers will be marked right (1 point) or wrong (0 points), with no partial credit.
  Scrap paper is available.
- **Part B** appears on pages 5 to 7 and consists of 6 short-response problems, worth 3 points each.
  Part C appears on pages 8 to 12 and consists of 5 long-response problems, worth 5 points each.
  For Parts B and C **work must be shown in the space provided.**
  Marks will be assigned for clarity of explanation / presentation.
- If you score at least 42/60 on this test **and** you provide all the information requested below, then you will be exempted from writing the Mathematics Placement Test at UNB and other participating universities.

If your score is at least 42/60, do you want your school to send your name and mark to the University of New Brunswick, for release to participating universities?

Circle your answer: Yes No

Check participating universities you wish to receive the mark:
_____ Mount Allison University
_____ Saint Mary's University
_____ University of New Brunswick

Print Name: ....
Last First Middle

Date of birth: Day: .... Month: .... Year: ....

Teacher's initials: ....

School: ....

Mark out of 60: [ ]

Signature: ....

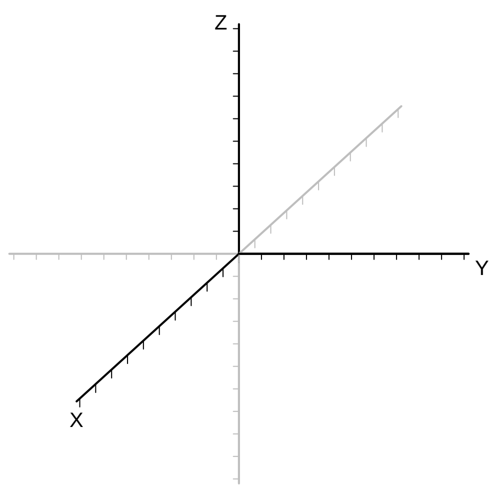
Name: ....

PART A: NO partial credit – 17 points.

Please do your rough work on the scrap paper provided.
Put your answer in the box or on the graph, as directed.

One point for each completely correct box or graph.

1. Sketch the plane given by $x - 2y + 3z = 6$.



2. Find the determinant of A.

$$A = \begin{pmatrix} -2 & -3 \\ 7 & 10 \end{pmatrix}$$

3. What is the solution of the system?

$$\begin{aligned} 5x + 3y &= 8 \\ 2x + 3y &= 14 \end{aligned}$$

$x =$

$y =$

4. Convert $250°$ to radian measure. (Give the exact value.)

5. Convert $\frac{3\pi}{5}$ radians to degrees.

6. Find the angle θ in the fourth quadrant with $\sin\theta = \frac{-\sqrt{3}}{2}$.

7. Determine the period of the function given by
$y = 3\tan 2(x - 70^\circ) + 1$.

8. Find the ***exact value*** of $\csc 210^\circ$.

9. List ***all*** asymptotes of the graph of $y = \tan(x - \frac{\pi}{4})$.

10. What is the smallest positive angle co-terminal with -710° ?

11. Simplify completely: $\cot\theta \sin\theta + \cos\theta$

12. If matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & 2 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 4 & 8 & 9 \\ 1 & 2 & 2 \\ 7 & 3 & 1 \end{bmatrix}$

then how many rows does the product AB have?

13. Given the following system of equations, what would be the corresponding matrix equation?

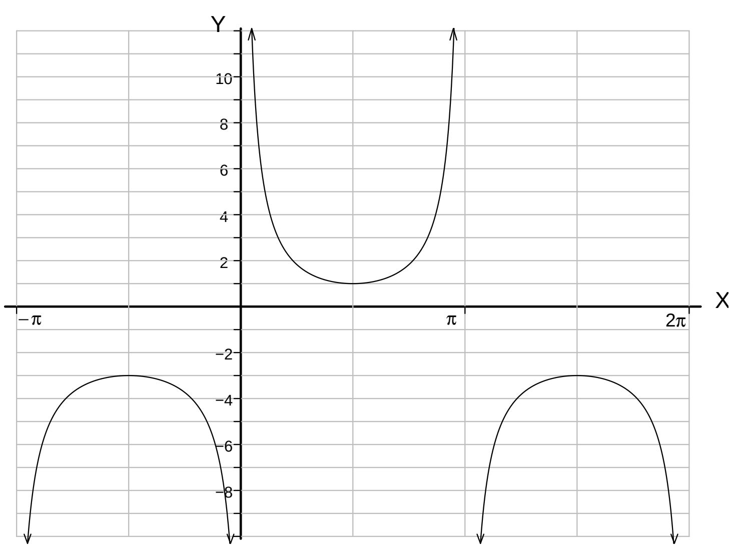
$$\begin{array}{rcrcrcr} 3x & + & 2y & + & z & = & -1 \\ 2x & + & 4y & - & 3z & = & -25 \\ 5x & - & y & + & 4z & = & 27 \end{array}$$

14. Find the ***exact value*** of $\sin^2(\frac{\pi}{3})$.

Express your answer as a simplified fraction.

15. If $f(x) = 2\sin x - 5$, find $f(30°)$.

16. Find the range of the function $y = f(x)$ graphed below.



17. Find ***all*** solutions of the equation $\cos\theta = \dfrac{-1}{\sqrt{2}}$.

PART B: Short response problems – 18 points.

Show your work clearly in the spaces provided.
Marks will be awarded for method, not for the final answer alone.

Each problem in this section is worth **3 points**.

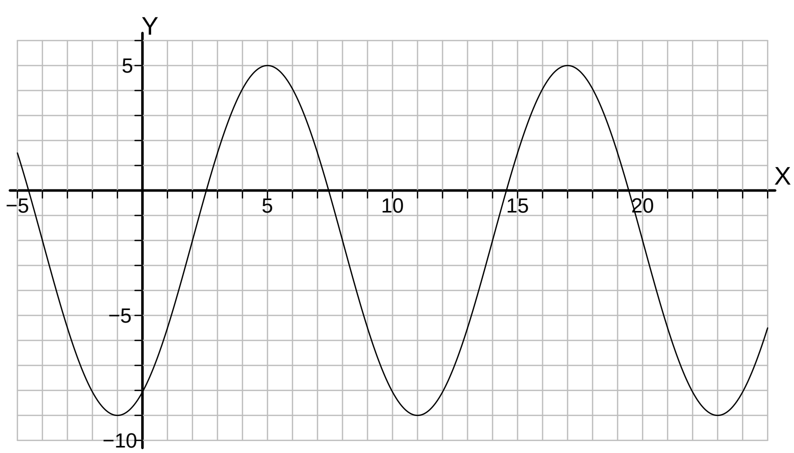
1. During this year's playoffs, a group of co-workers selected teams as part of a hockey pool. Each team receives a specific number of points for goals, assists and shutouts.

   Billy's team had 15 goals, 35 assists and 1 shutout. Subin's team had 20 goals, 15 assists and 3 shutouts. Monica's team had 10 goals, 25 assists and 2 shutouts. The total points earned by Billy's, Subin's and Monica's teams were 85, 90 and 65 respectively.

   Write down 3 equations which could be used to determine the points awarded for each goal, assist and shutout. Use G to represent the points awarded for a goal, A to represent points awarded for an assist and S to represent points awarded for a shutout.

   Do not solve the equations!

2. Write down an equation for the graph, in terms of either the sine or cosine function, for x measured in degrees.

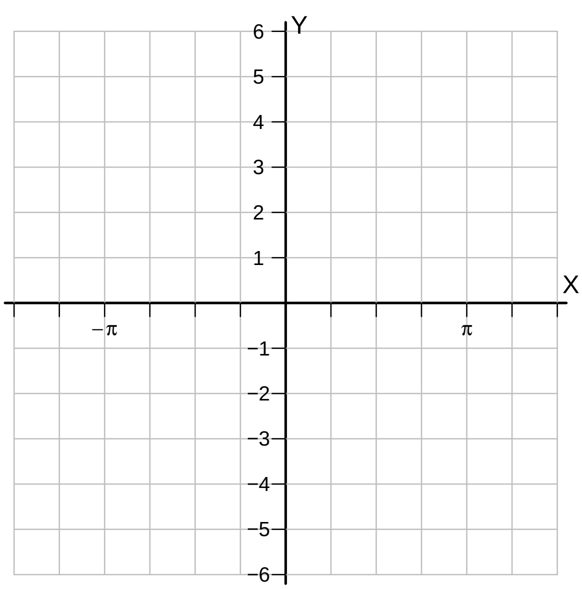


3. Solve the system of equations below using the inverse matrix method.

$$\begin{array}{rcrcr} 3x & + & 4y & = & 1 \\ 5x & + & 10y & = & -5 \end{array}$$

4. Sketch the graph of the equation given below, indicating basic shape, key points and asymptotes.

$$y - 2 = \sec(x + \frac{\pi}{2})$$



5. Simplify the rational expression below and state restrictions.

$$\frac{x-3}{x+4} + \frac{8x-17}{x^2+x-12}$$

6. Find the exact value of $\cos\frac{7\pi}{12}$. (Remember to show your work.)

PART C: Long response problems – 25 points.

Show your work clearly in the spaces provided.
Marks will be awarded for <u>method</u>, not for the final answer alone.

Each problem in this section is worth **5 points**.

1. Solve the system of equations.

Show work. A calculator or trial-and-error solution is not acceptable!

$$\begin{array}{rcrcrcr} 2x & + & 3y & + & z & = & 14 \\ 5x & + & y & + & 2z & = & 27 \\ x & + & 5y & - & 4z & = & -3 \end{array}$$

2. Find the exact value of the following expression.

   Simplify your answer completely and rationalize the denominator.

$$\frac{3\cos 45^\circ}{1 + 4\sin 240^\circ}$$

3. Prove each identity.

(a) $$\frac{\cos 2\theta}{\cos\theta + \sin\theta} = \cos\theta - \sin\theta.$$

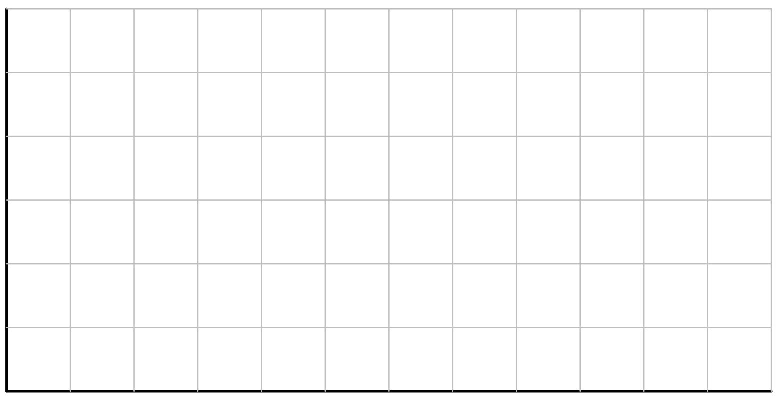
(b) $$\frac{\sin\theta + \cos\theta\ \cot\theta}{\cot\theta} = \sec\theta.$$

4. (a) Solve $\cos 2\theta = -\frac{1}{2}$ for $\theta \in [0, 2\pi]$.

(b) Solve $\tan^2 \theta - \tan \theta = 0$ for $\theta \in R$.

5. Sarah is sitting in an inner-tube in a wave pool. Her height above the bottom of the pool varies sinusoidally with time. At time $t = 2$ seconds Sarah is at the top of a wave, 3.5 metres from the bottom of the pool. At $t = 4$ seconds she is at the bottom of a wave, 2.5 metres from the bottom of the pool.

   (a) Mark appropriate scales on the axes below and sketch a graph that represents Sarah's height above the bottom of the pool as a function of time. Show two complete periods.



   (b) Write down an equation in t and h that describes the graph of part (a).

   (c) Use the equation from part (b) to determine Sarah's height above the bottom of the pool at time $t = 12.5$ seconds.

Math 122 – Formula Sheet

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$