Trigonometry and 3-Space

Sample Final Examination: Mathematics 122 version 2009-B

This examination paper was prepared by a committee of NB high school teachers,
as part of a workshop held at the University of New Brunswick, March 12 – 13, 2009.

Time: 2 hours.
Calculators: Scientific calculator required.
Graphing calculator permitted, at the teacher's discretion.
Formulae: see the last page.

- Please check that there are 14 pages, including this page.
- There are three parts to the test, Parts A (14 points), B (21 points) and C (25 points).
  Total points: 60.
- **Part A** appears on pages 2 to 4 and consists of 14 short problems.
  Write your answers clearly in the boxes or on the graphs provided.
  Answers will be marked right (1 point) or wrong (0 points), with no partial credit.
  Scrap paper is available.
- **Part B** appears on pages 5 to 8 and consists of 7 short-response problems, worth 3 points each.
  Part C appears on pages 9 to 13 and consists of 5 long-response problems, worth 5 points each.
  For Parts B and C **work must be shown in the space provided.**
  Marks will be assigned for clarity of explanation / presentation.
- If you score at least 42/60 on this test **and** you provide all the information requested below, then you will be exempted from writing the Mathematics Placement Test at UNB and other participating universities.

If your score is at least 42/60, do you want your school to send your name and mark to the University of New Brunswick, for release to participating universities?

Circle your answer: Yes No

Check participating universities you wish to receive the mark:
_____ Mount Allison University
_____ Saint Mary's University
_____ University of New Brunswick

Print Name: ..... (Last / First / Middle)

Date of birth: Day: ..... Month: ..... Year: .....

School: .....

Signature: .....

Teacher's initials: .....

Mark out of 60:

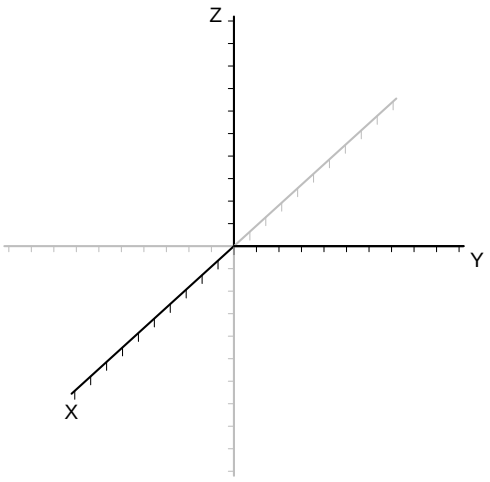
Name: .....

PART A: NO partial credit – 14 points.

Please do your rough work on the scrap paper provided.
Put your answer in the box or on the graph, as directed.

One point for each completely correct box or graph.

1. Sketch the plane $2x + 3y - 4z = 12$.



2. Convert the matrix equation to a system of equations.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

3. For what value of d will the matrix below have determinant equal to zero?

$$\begin{pmatrix} 2 & 3 \\ 4 & d \end{pmatrix}$$

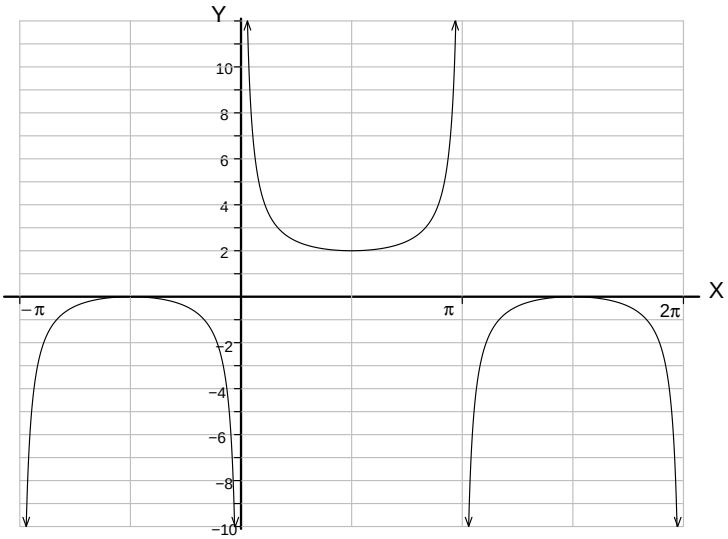
4. Express $140°$ as a radian measure. (Give the exact value.)

5. Express $\dfrac{5\pi}{3}$ radians in degrees.

6. Simplify $\dfrac{\sec\theta}{\tan\theta}$.

7. Determine the amplitude of the graph of
$y = 2\cos 3(x + 60^\circ) - 4$.

8. Find the range of the function $y = f(x)$ graphed below.



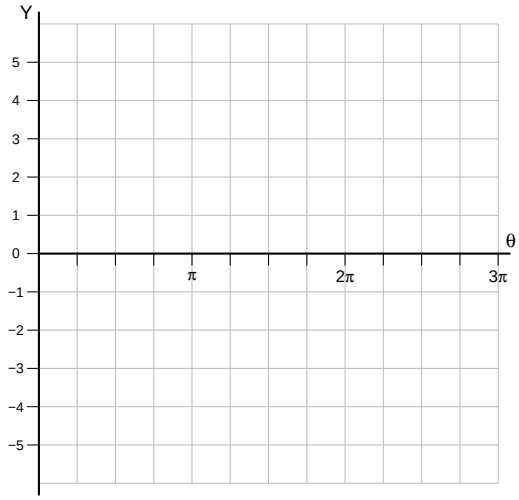
9. What angle θ, in the second quadrant, is a solution of this equation?

$\sin\theta = \dfrac{1}{2}$

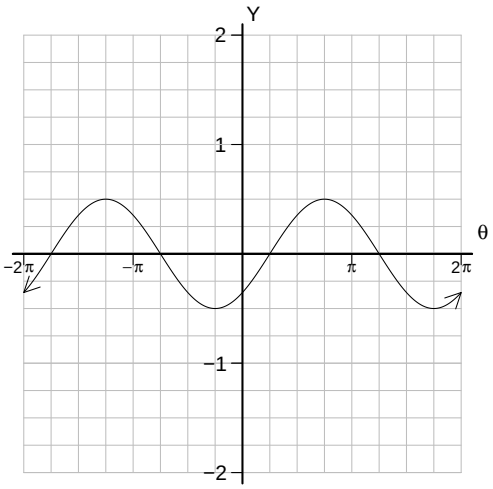
10. Determine the location of ***all*** asymptotes for the graph of

$y = \tan x + 1$.

11. Sketch the graph of $y = \cot(\theta)$.

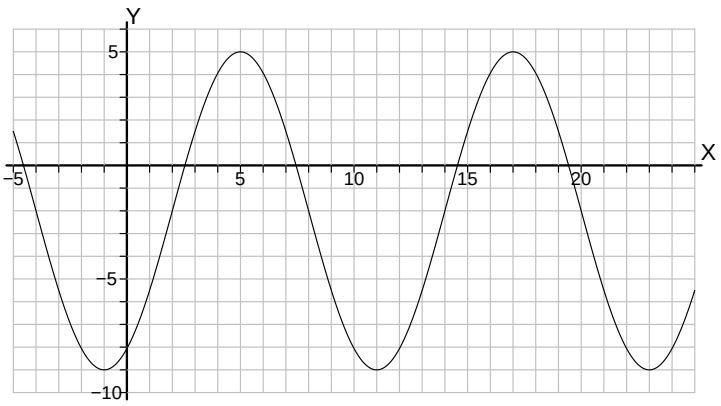


12. Write down an equation for the sinusoidal graph shown below.



13. Write $y = 2\sin(\theta)$ in terms of the cosine function.

14. Determine the period of the function graphed below, with x measured in degrees.



PART B: Short response problems – 21 points.

Show your work clearly in the spaces provided.
Marks will be awarded for <u>method</u>, not for the final answer alone.

Each problem in this section is worth **<u>3 points</u>**.

1. Given the matrix $N = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}$

   (a) Calculate the determinant of matrix N.

   (b) Find the inverse of matrix N.

   (c) Prove that the matrix found in (b) is the inverse of matrix N.

2. Which of these points are on the plane $2x + 4y + z = 20$?

   $$A = (-4, 2, 20) \qquad B = (1, 5, -1) \qquad C = (10, -2, 8)$$

   Explain how you know.

3. Simplify the rational expression below and state restrictions.

$$\frac{x^2-9}{2x^2+8x} \div \frac{x^2-6x+9}{x^2+x-12}$$

4. Twenty people attended a family reunion at Canada's Wonderland, where admission prices are \$27.50 for adults, \$21.50 for seniors and \$18.75 for children. The total cost of admissions was \$433.00 and there were twice as many children as adults.

   Write three equations that could be solved to determine the number of each type of ticket purchased. Use A to represent the number of adult tickets purchased, S to represent the number of senior tickets purchased and use C to represent the number of children's tickets purchased.

   Do <u>not</u> solve the equations!

5. Find ***all*** values of x for which $\cos 2x = \dfrac{\sqrt{2}}{2}$.

6. Prove the identity $\csc x - \cos x \cot x = \sin x$.

7. Use a compound angle identity to find the exact value of $\sin \frac{11\pi}{12}$.
(Remember to show your work.)

PART C starts on the next page.

PART C: Long response problems – 25 points.

Show your work clearly in the spaces provided.
Marks will be awarded for <u>method</u>, not for the final answer alone.

Each problem in this section is worth **<u>5 points</u>**.

1. Solve the system of equations.

A calculator or trial-and-error solution is <u>not</u> acceptable!

$$\begin{aligned} a + b + 2c &= 0 \\ 2a - 5b - 4c &= 17 \\ 3a - 2b + 6c &= 15 \end{aligned}$$

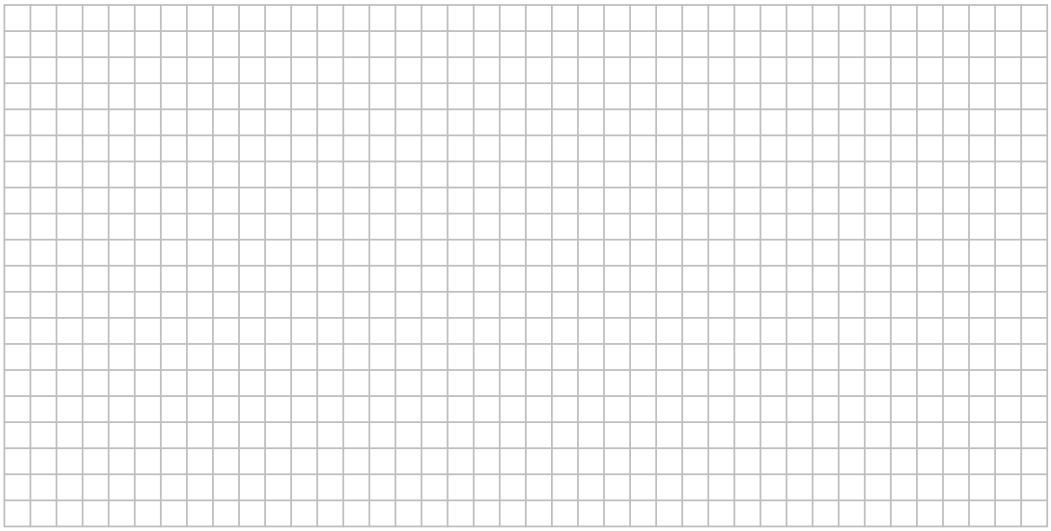
2. Find the exact value of the following expression.

   Simplify your answer completely and rationalize the denominator.

$$\frac{\cos^2 30^\circ}{\sin 225^\circ + \cos 180^\circ}$$

3. A water wheel with diameter 8 metres takes 16 minutes for 1 revolution. As the wheel begins to turn, a nail on its rim is at the lowest point, 1 metre below the water surface.

   (a) Mark appropriate axes, with scales, and draw a graph to represent height of the nail above (or below) water level during two revolutions of the wheel.



   (b) Write an equation describing the relationship between height (h in m) and time since the wheel started turning (t in minutes).

   (c) Use your graph to estimate height of the nail after 6 minutes.
   (Show how you used the graph.)

4. Prove the following identities.

(a) $2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = \sin 2\theta.$

(b) $\dfrac{\cos(B-A)}{\cos B\ \sin A} = \tan B + \cot A.$

5. Find ***all*** solutions of the equation below (for θ measured in radians).

$$\frac{\tan\theta \;-\; \sin 2\theta}{\tan\theta \;+\; \sin 2\theta} \;=\; 0$$

Math 122 – Formula Sheet

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$